

- HAUPTMAN, H. & FORTIER, S. (1977). *Acta Cryst.* **A33**, 575–580.
- HEINERMAN, J. J. L. (1978). *Proceedings of the International Symposium on Direct Methods in Crystallography*, Buffalo, New York, edited by H. HAUPTMAN, pp. 177–197.
- HEINERMAN, J. J. L., KRABBENDAM, H. & KROON, J. (1977). *Acta Cryst.* **A33**, 873–878.
- PUTTEN, N. VAN DER & SCHENK, H. (1977). *Acta Cryst.* **A33**, 856–858.
- SCHENK, H. (1975). *Acta Cryst.* **A31**, S14.

*Acta Cryst.* (1979). **A35**, 381–387

## Application of Quartet and Quintet Invariants in Phase Determination

BY NIEK VAN DER PUTTEN AND HENK SCHENK

*Laboratory for Crystallography, University of Amsterdam, Nieuwe Achtergracht 166, Amsterdam, The Netherlands*

(Received 2 October 1978; accepted 9 November 1978)

### Abstract

Recent derivations of probability expressions for quartet and quintet structure invariants allow the reliable estimation of the phase sums of quartets and quintets in the range 0 to  $\pi$ . A new quartet figure of merit, ENQUAC and a new quintet figure of merit, ENQUIC, based on these estimates are described, which are particularly useful in non-centrosymmetric symmorphic and polar space groups. An adapted tangent-refinement procedure employing selected triplets with a phase sum of 0, together with the quartet and quintet phase estimates, enables enantiomorph-specific phase refinement. The way in which the figures of merit and the refinement technique can be used in various practical procedures is demonstrated and applications to two structures in space group  $P1$  are presented.

### Introduction

Although more and more structures of moderate complexity (60 to 80 independent atoms) are solved by direct methods, in non-centrosymmetric symmorphic and polar space groups, the application of the triplet relation fails for numerous structures: parts of the phase determination, which are easy to carry out in other space groups, give rise to serious problems, such as:

(i) The enantiomorph definition; in the space groups  $P1$ ,  $P2$ ,  $P2_1$ ,  $C2$  and  $Cc$  it is a difficult procedure, because it is not possible to select a starting reflection, which is enantiomorph-sensitive ( $\Phi \simeq \pm \pi/2$ ).

(ii) The tangent refinement; occasionally the resulting phases are centrosymmetric, even starting with a correct phase set.

(iii) The various figures of merit, based on triplets; they do not discriminate as consistently between correct and incorrect phase sets as they do in non-symmorphic and non-polar space groups. Some of these difficulties have been dealt with by Schenk (1972), who indicated that they can be ascribed to the influence of the space group symmetry and the properties of the  $\sum_2$  relation.

These problems do not exist if the actual values of the phase sums,

$$\Phi_3 = \Phi_H + \Phi_K + \Phi_{-H-K}, \quad (1)$$

could be used instead of assuming all  $\Phi_3$  to be zero. Of course the  $\Phi_3$  values cannot be evaluated from  $|E|$  magnitudes alone, and in practice only  $|\Phi_3|$  values can be calculated, of which the quality is not yet sufficiently good to define reliable enantiomorph-specific procedures (e.g.  $B_{3,0}$  formula, Karle & Hauptman, 1958; TPROD formula, Hauptman, Fischer, Hancock & Norton, 1969; MDKS formula, Fischer, Hancock & Hauptman, 1970; and the strengthened triplet formula, Giacovazzo, 1977).

Test results of recent expressions for quartets and quintets proved that the absolute values  $|\Phi_4|$  and  $|\Phi_5|$  of their phase sums,

$$\Phi_4 = \Phi_H + \Phi_K + \Phi_L + \Phi_{-H-K-L} \quad (2)$$

and

$$\Phi_5 = \Phi_H + \Phi_K + \Phi_L + \Phi_M + \Phi_{-H-K-L-M}, \quad (3)$$

can be estimated with the required accuracy for enantiomorph-specific procedures. Most of these results are described in papers by Hauptman (1975), van der Putten & Schenk (1976), Schenk & van der Putten (1977), van der Putten & Schenk (1977) and Gilmore (1977). More evidence will be given in this paper.

The main problem left when applying estimated phase sums in structure determination arises from the

fact that only the absolute values of  $\Phi_4$  and  $\Phi_5$  can be estimated. The choice between  $-\Phi_4$  and  $+\Phi_4$  has to be made in the practical application itself and the same applies to  $-\Phi_5$  and  $+\Phi_5$ . How this is achieved in the enantiomorph-specific figures of merit ENQUAC (based on quartets) and ENQUIC (based on quintets) and in an enantiomorph-specific refinement procedure is the main subject of this paper. Finally, the role of these procedures in phase determination will be discussed and illustrated.

#### Estimates of quartet phase sums $|\Phi_4|$

Hauptman (1975) has derived a probability expression to estimate the phase sum  $|\Phi_4|$  of a quartet structure invariant given the  $E$  magnitudes of the reflections  $H$ ,  $K$ ,  $L$  and  $H + K + L$  and those of the three cross-reflections  $H + K$ ,  $H + L$ ,  $K + L$ :

$$P(1|7) = P(\Phi_4 | |E_H|, |E_K|, |E_L|, |E_{H+K+L}|, |E_{H+K}|, |E_{H+L}|, |E_{K+L}|) \\ = c \exp(-4E4 \cos \Phi_4) \prod_3 I_0(2N^{-1/2} |E_{H+K}| Y_{H+K}) \quad (4)$$

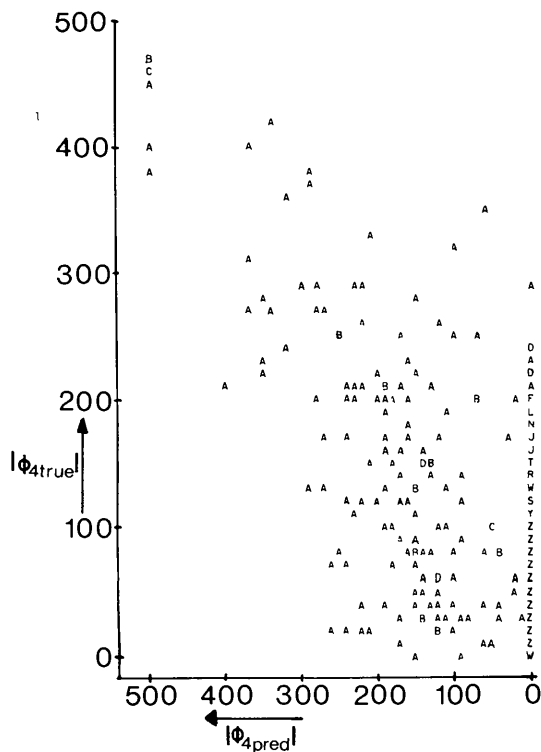


Fig. 1. Graph of  $|\Phi_{4(\text{true})}|$  against  $|\Phi_{4(\text{pred})}|$  in millicycles (1000 millicycles =  $2\pi$ ) predicted with (4) for 825 quartets of a 30 atom structure. The number of quartets with the same  $|\Phi_{4(\text{true})}|$  and  $|\Phi_{4(\text{pred})}|$  are represented by capital letters: A means 1, B 2, C 3 etc.

in which  $E4 = N^{-1} |E_H E_K E_L E_{H+K+L}|$ ,  $c$  is a suitable normalizing constant,  $Y_{H+K} = (E_H^2 E_K^2 + E_L^2 E_{H+K+L}^2 + 2 |E_H E_K E_L E_{H+K+L}| \cos \Phi_4)^{1/2}$ . This distribution gives maxima (modes) within the range  $0 \leq |\Phi_4| \leq \pi$ . It has been applied to two structures in space group  $P1$ .

For a 30 atom structure (Kanters & Van Veen, 1973) 825 quartets were calculated for the 200 strongest reflections down to a limit  $E4 = 0.7$ . In Fig. 1 the mode of the phase sums  $|\Phi_4|$  predicted by (4) are plotted against the true phase sums  $|\Phi_4|$ . The overall difference  $\langle ||\Phi_{4(\text{pred})}| - |\Phi_{4(\text{true})}| \rangle$  is 73 millicycles.

In Fig. 2 a similar plot is given for 940 quartets which have been calculated within the group of the 500 strongest idealized  $E$  magnitudes of a 44 atom structure (Mairesse & Dracke, 1978), down to a limit value of  $E4 = 0.7$ . For these quartets the mean difference  $\langle ||\Phi_{4(\text{pred})}| - |\Phi_{4(\text{true})}| \rangle = 78$  millicycles. More test results can be found in Schenk (1976) and Gilmore (1977). On the basis of this material and extensive tests on randomly generated structures (van der Putten, 1978) we could conclude that  $P(1|7)$  is not reliable for small structures ( $N$  smaller than approximately 10) and large ones ( $N$  larger than approximately 200). For small structures the modes of  $P(1|7)$  are for the greater part wrongly found in the range  $\pi/3 < |\Phi_4| \leq \pi$  and for large structures there is a tendency for very broad distributions with mode  $|\Phi_4| = 0$ .

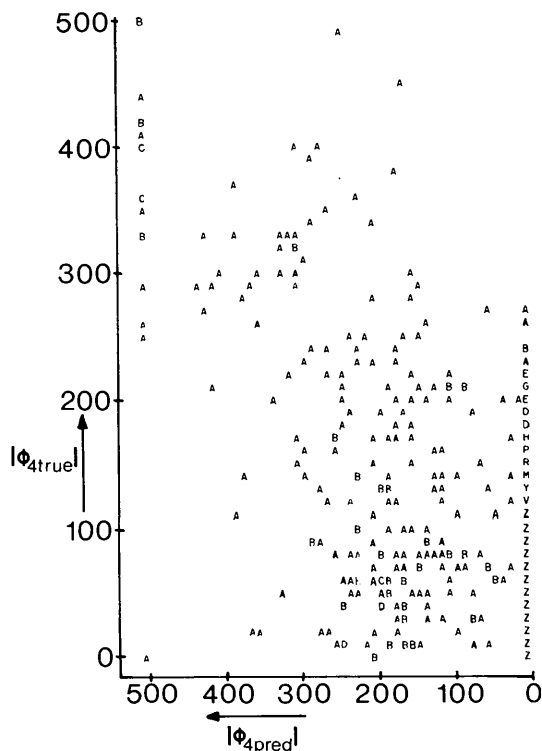


Fig. 2. Graph of  $|\Phi_{4(\text{true})}|$  against  $|\Phi_{4(\text{pred})}|$  predicted with (4) for 940 quartets of a 44 atom structure.

Figs. 1 and 2 show that the number of predicted phase sums of 0 and  $\pi$  is overdetermined, which will cause problems in the applications, because estimates of 0 and  $\pi$  do not discriminate between the enantiomorphs. On the other hand the  $|\Phi_4|$  values with modes other than 0 or  $\pi$  are reliably estimated but are too few in number. Therefore the following  $|\Phi_4|$  estimates are used in our enantiomorph-specific procedures:

$$|\Phi_4|_{\text{used}} = ||\Phi_4|_{\text{mode}} - a\sigma| \quad (5)$$

in which

$$a = 0 \quad \text{for } 0 < |\Phi_4|_{\text{mode}} < \pi$$

$$\text{and } a = 0.33 \quad \text{for } |\Phi_4|_{\text{mode}} = 0 \text{ or } \pi,$$

and  $\sigma$  is the standard deviation given by

$$\sigma = \left( \int_0^\pi (|\Phi_4| - |\Phi_4|_{\text{mode}})^2 P(1|7) d\Phi_4 \right)^{1/2}. \quad (6)$$

Experiments with enantiomorph-specific procedures which will be described in the sequel showed that the estimates from (5) give better results than modes (suggested by Gilmore, 1977) or means (suggested by Hauptman, 1978 and Gilmore, 1978).

### Enantiomorph-specific figure of merit ENQUAC

On the basis of the estimates of the phase sums  $|\Phi_4|$  described in the previous section an enantiomorph-specific figure of merit ENQUAC can be formulated:

$$\text{ENQUAC} = \sum_i W_i |\Phi_H + \Phi_K + \Phi_L + \Phi_{-H-K-L} + S_i |\Phi_4||, \quad (7)$$

in which  $\Phi_H$ ,  $\Phi_K$ ,  $\Phi_L$  and  $\Phi_{-H-K-L}$  are determined in a direct phasing procedure,  $W_i = \sigma^{-1}$  with  $\sigma$  defined by (6) and  $S_i = \pm 1$  to be chosen such that  $\Phi_H + \Phi_K + \Phi_L + \Phi_{-H-K-L} + S_i |\Phi_4|$  is nearest to zero.

If the phasing procedure gives the correct values for  $\Phi_H$ ,  $\Phi_K$ ,  $\Phi_L$  and  $\Phi_{-H-K-L}$  and if the  $|\Phi_4|$  estimation gives a correct value for  $|\Phi_4|$ , the resulting ENQUAC criterion will be equal to zero. The nearest centrosymmetric phases for  $\Phi_H$ , etc. give rise to a much higher ENQUAC value. It follows that ENQUAC should be enantiomorph-specific. In practice the values of  $\Phi_H$ ,  $\Phi_K$ ,  $\Phi_L$  and  $\Phi_{-H-K-L}$  and  $|\Phi_4|$  are afflicted with errors. However, as long as the errors are more random than systematic, the ENQUAC criterion will still be enantiomorph-specific and it is expected that the smallest ENQUAC value corresponds to the set of correct phases of the structure.

The ENQUAC figure of merit can be used in both multiresolution tangent refinement and symbolic addition procedures. In the first procedure for each solution an ENQUAC value can be calculated. To bring these onto

a relative scale it is necessary to use instead of (7) the related criterion:

$$\text{ENQUAC} = \frac{\sum_i W_i |\Phi_H + \Phi_K + \Phi_L + \Phi_{-H-K-L} + S_i |\Phi_4||}{\sum W_i}, \quad (8)$$

which enables an easy comparison of the ENQUAC values. In the second case the  $\Phi_H$ ,  $\Phi_K$ ,  $\Phi_L$  and  $\Phi_{-H-K-L}$  are expressed in the symbols  $X_j$  and thus the figure of merit (7) can be rewritten as:

$$\text{ENQUAC}(X_1, X_2, \dots, X_n) = \frac{\sum_i W_i |\sum_j A_{ij} X_j + S_i |\Phi_4||}{\sum W_i}. \quad (9)$$

Then for sets of numerical values of  $X_j$  the ENQUAC figure of merit can be evaluated. Rescaling is not necessary.

Starting from a set of parameter values in (9) it is possible to use an iterative least-squares procedure to refine the  $X_j$  values. The function to be minimized is:

$$R(X_j) = \sum_i W_i |\sum_j A_{ij} X_j + S_i |\Phi_4||^2. \quad (10)$$

### The negative-quartet criterion NEGQAC

In recent years several proposals have been made for a figure of merit based on negative quartets:

$$\Phi_H + \Phi_K + \Phi_L + \Phi_{-H-K-L} \simeq \pi,$$

for large  $E_4$  and small  $|E_{H+K}|$ ,  $|E_{H+L}|$  and  $|E_{K+L}|$ . On the basis of the first formulation of this phase relation (Hauptman, 1974), Schenk (1974) formulated a negative-quartet criterion as:

$$\text{NQC} = \sum_i W_i |\Phi_H + \Phi_K + \Phi_L + \Phi_{-H-K-L} - \pi|, \quad (11)$$

in which

$$\left. \begin{aligned} W_i &= E4(2.7 - |E_{H+K}| - |E_{H+L}| - |E_{K+L}|) \\ \text{and} \\ W_i &= E4(1.0 - |E_{H+K}| - |E_{H+L}|) \end{aligned} \right\}, \quad (12)$$

for quartets with three and two known cross-reflections respectively.

On the basis of a theory of Giacovazzo (1975), Schenk (1975) reformulated the weights (12):

$$\left. \begin{aligned} W_i &= E4(2 - |E_{H+K}|^2 - |E_{H+L}|^2 - |E_{K+L}|^2) \\ \text{and} \\ W_i &= E4(1 - |E_{H+K}|^2 - |E_{H+L}|^2) \end{aligned} \right\}, \quad (13)$$

for quartets with three and two known cross-reflections respectively.

DeTitta, Edmonds, Langa & Hauptman (1975) later published the NQUEST criterion:

$$\text{NQUEST} = \frac{\sum_i E4 |\Phi_H + \Phi_K + \Phi_L + \Phi_{-H-K-L}|}{\sum E4}, \quad (14)$$

with the summation over the negative quartets.

These negative quartets are selected by strict thresholds for all seven magnitudes involved. All negative quartets accepted in the summation have a weight exclusively determined by the value of  $E_4$  and independent of the values of  $|E_{H+K}|$ ,  $|E_{H+L}|$  and  $|E_{K+L}|$ . In spite of the criticism of DeTitta, Edmonds, Langs & Hauptman (1975) on the negative-quartet criterion NQC the incorporation of weights based on all seven magnitudes involved gives rise to better results. A further improvement of figures of merit based on negative quartets will involve the use of the estimated  $|\Phi_4|$  values. This figure of merit can be written as:

$$\text{NEGQAC} = \sum_i W_i |\Phi_H + \Phi_K + \Phi_L + \Phi_{-H-K-L} + S_i |\Phi_4||, \quad (15)$$

in which  $S_i (= \pm 1)$  is chosen such that  $\Phi_H + \Phi_K + \Phi_L + \Phi_{-H-K-L} + S_i |\Phi_4|$  is closest to zero and the summation is restricted to those quartets which are predicted to be negative (e.g.  $|\Phi_4| > 0.7\pi$ ). Since it can be seen from the tests that the  $|\Phi_4|$  values are estimated with a random error, NEGQAC is expected to be a better criterion than NQC and NQUEST.

NEGQAC is calculated with a selection of the terms included in ENQUAC. Since our experiences with related figures of merit indicate that the more terms that are included the more reliable the figure of merit will be, we expect ENQUAC to be more selective than NEGQAC.

#### Estimates of quintet phase sums $|\Phi_5|$

On the basis of the exponential probability expression for quintet phase sums (3) (Hauptman & Fortier, 1977), van der Putten & Schenk (1977) derived a probability expression in the exponential Bessel function form given the  $E$  magnitudes of the five main reflections  $H, K, L, M$  and  $H + K + L + M$  and those of the ten cross-reflections  $H + K, H + L, H + M, K + L + M, K + L, K + M, H + L + M, L + M, H + K + M$  and  $H + K + L$ :

$$P(1|15) = C' \exp \left\{ \left( 6 - \sum_{10 \text{ terms}} E_{H+K}^2 \right) 2E_5 \cos \Phi_5 \right\} \\ \times \prod_{10 \text{ terms}} I_0(2N^{-3/4} |E_{H+K}| Y_{H+K}), \quad (16)$$

in which  $C'$  is a suitable normalizing constant,

$$\sum_{10 \text{ terms}} E_{H+K}^2 = E_{H+K}^2 + E_{H+L}^2 + \dots + E_{H+K+L}^2,$$

$$E_5 = N^{-3/2} |E_H E_K E_L E_M E_{H+K+L+M}|, \\ Y_{H+K} = \{ 4^{-1} E_H^2 E_K^2 (E_{L+M}^2 + E_{H+K+M}^2 + E_{H+K+L}^2 - 2)^2 \\ + |E_H E_K E_L E_M E_{H+K+L+M}| (E_{L+M}^2 \\ + E_{H+K+M}^2 + E_{H+K+L}^2 - 2) \cos \Phi_5 \\ + E_L^2 E_M^2 E_{H+K+L+M}^2 \}^{1/2}.$$

This expression also gives modes in the range  $0 \leq |\Phi_5| \leq \pi$ . The first test results (van der Putten & Schenk, 1977; Gilmore, 1978) showed good agreement between the estimated and the true phase sums  $|\Phi_5|$  and the first authors concluded that the  $|\Phi_5|$  estimates could be used in enantiomorph-specific procedures. Although the  $|\Phi_5|$  estimates have somewhat larger variances than the  $|\Phi_4|$  estimates, the fact that a much larger fraction of the quintets is enantiomorph-sensitive will compensate for this. For the same reasons as for the quartets the  $|\Phi_5|$  the values used in most of the practical procedures are given by:

$$|\Phi_5|_{\text{used}} = ||\Phi_5|_{\text{mode}} - a\sigma|, \quad (17)$$

in which

$$\sigma = \left( \int_0^\pi (|\Phi_5| - |\Phi_5|_{\text{mode}})^2 P(1|15) d\Phi_5 \right)^{1/2}, \quad (18)$$

$$a = 0 \quad \text{for } 0 < |\Phi_5|_{\text{mode}} < \pi$$

$$\text{and } a = 0.33 \quad \text{for } |\Phi_5|_{\text{mode}} = 0 \text{ or } \pi.$$

#### Enantiomorph-specific figure of merit ENQUIC

In analogy with the quartet criterion ENQUAC we define an enantiomorph-specific figure of merit ENQUIC:

$$\text{ENQUIC} = \sum_i W_i |\Phi_H + \Phi_K + \Phi_L + \Phi_M + \Phi_{-H-K-L-M} + S_i |\Phi_5||, \quad (19)$$

in which  $|\Phi_5|$  is the estimated phase sum,  $W_i = \sigma^{-1}$ ,  $\sigma$  = the standard deviation defined by (18),  $S_i = \pm 1$  to be chosen such that  $\Phi_H + \Phi_K + \Phi_L + \Phi_M + \Phi_{-H-K-L-M} + S_i |\Phi_5|$  is closest to zero.

An identical reasoning as for ENQUAC shows that ENQUIC is enantiomorph-specific. The figure of merit can be used in the same way in multiresolution tangent refinement and symbolic addition. In the last case the phase sum  $\Phi_H + \Phi_K + \Phi_L + \Phi_M + \Phi_{-H-K-L-M}$  is expressed in the symbols  $X_j$  and thus the figure of merit (19) can be rewritten as:

$$\text{ENQUIC}(X_1, X_2, \dots, X_n) = \sum_i W_i \left| \sum_j A_{ij} X_j + S_i |\Phi_5| \right|. \quad (20)$$

For sets of numerical values of  $X_j$  the ENQUIC figure of merit can then be evaluated. The sets of  $X_j$  values with lowest ENQUIC may be refined by iterative least squares. The function to be minimized is:

$$R(X_j) = \sum_i W_i \left| \sum_j A_{ij} X_j + S_i |\Phi_3| \right|^2. \quad (21)$$

### Enantiomorph-specific phase extension and refinement

It is also possible to use the estimated  $|\Phi_4|$  and  $|\Phi_5|$  values in an enantiomorph-specific phase extension and refinement procedure. Initially we developed separate procedures for quartets and quintets, but in the end a method using the complete set of quartets and quintets, combined with those triplets estimated to be reliably near zero ( $\Phi_3 = 0$ ), proved to be most successful. The procedure consists of two parts, the phase extension and the phase refinement. The extension part employs the strongest triplets ( $\Phi_3 = 0$ ) and the positive and negative quartets and quintets only ( $|\Phi_4|_{\text{mode}} = 0, \pi$ ;  $|\Phi_5|_{\text{mode}} = 0, \pi$ ). The new phases are calculated by means of a variant of the usual tangent formula:

$$\tan \Phi_H = \frac{\text{stri} + \text{squa} + \text{squi}}{\text{ctri} + \text{cqua} + \text{cqui}}, \quad (22)$$

with

$$\text{stri} = \sum_K W_3 \sin(\Phi_K + \Phi_{H-K}),$$

$$\text{squa} = \sum_K \sum_L W_4 \sin(\Phi_K + \Phi_L + \Phi_{H-K-L} + |\Phi_4|_{\text{mode}}),$$

$$\text{squi} = \sum_K \sum_L \sum_M W_5 \sin(\Phi_K + \Phi_L + \Phi_M + \Phi_{H-K-L-M} + |\Phi_5|_{\text{mode}}),$$

and corresponding expressions for the cosine terms in the denominator.

Then the complete set of known phases is refined with an adapted form of the above tangent formula employing the strongest triplets and all quartets and quintets:

$$\tan \Phi_H = \frac{\text{sintri} + \text{sinqua} + \text{sinqui}}{\text{costri} + \text{cosqua} + \text{cosqui}}, \quad (23)$$

with

$$\text{sintri} = \sum_K W_3 \sin(\Phi_K + \Phi_{H-K}),$$

$$\text{sinqua} = \sum_K \sum_L W_4 \sin(\Phi_K + \Phi_L + \Phi_{H-K-L} + S_4 |\Phi_4|),$$

$$\text{sinqui} = \sum_K \sum_L \sum_M W_5 \sin(\Phi_K + \Phi_L + \Phi_M + \Phi_{H-K-L-M} + S_5 |\Phi_5|),$$

and corresponding expressions for  $\text{costri}$ ,  $\text{cosqua}$  and  $\text{cosqui}$ . In this expression the  $W_i$ 's are proportional to the reciprocal standard deviations of the respective structure invariants,  $|\Phi_4|$  and  $|\Phi_5|$  are given by (5) and (17) respectively, and  $S_4(\pm 1)$  and  $S_5(\pm 1)$  are chosen such that  $-\Phi_H + \Phi_K + \Phi_L + \Phi_{H-K-L} + S_4 |\Phi_4|$  and  $-\Phi_H + \Phi_K + \Phi_L + \Phi_M + \Phi_{H-K-L-M} + S_5 |\Phi_5|$  respectively are closest to zero. As a result of the use of the  $|\Phi_4|$  and  $|\Phi_5|$  values this adapted tangent refinement is able to maintain the enantiomorph. The use of the triplets does not affect this property, because the triplets are selected such that only small deviations from  $\Phi_3 = 0$  are included in the calculations. The quartet and quintet parts of (23) are related to the tangent procedure of Sint & Schenk (1975), which proved successful for phase refinement in large molecules with large sets of starting phases. Lessinger (1976) described possible future strategies for *MULTAN*. He found that application of the normal tangent formula drives the true phases to values which make the triplet relationships among the largest  $E$  values very consistent and the invariants much more narrowly distributed about zero than they in fact are. Therefore he proposed an extended tangent formula containing triplets and quartets, in particular the negative quartets. The reason for the use of the negative quartets was that these invariants would change the relations which the phase of the largest  $E$  values must satisfy and would constrain the phase development. Our results suggest that the use of triplets, quartets and quintets together with their estimated phase sum gives a better clue to this problem than the procedure proposed by Lessinger. The main reasons for this are:

(i) There is great abundance of quartets and in particular of quintets within the group of reflections with the largest  $E$  values. However, the number of negative quartets within that group is very small.

(ii) A large number of the phase sums of quartets and quintets may be estimated reliably. This excess of information enables a rigorous selection of triplets, quartets and quintets in order to maintain the enantiomorph-specific properties of the refinement procedure.

Of course the method has the limitation that it is not suitable for very large structures, because the reliability of their estimated phase sums and especially of their enantiomorph-sensitive phase sums is too low.

### ENQUAC, ENQUIC and the enantiomorph-specific refinement in the symbolic addition program system *SIMPEL*

The enantiomorph-specific procedures can be easily incorporated in the interactive direct methods program system *SIMPEL* (Overbeek, van der Putten, Olthof & Schenk, 1977). *SIMPEL* has a modular structure, in

which the major routines are: generation of (sem-) invariants, setting up the starting set, phase extension, evaluation of figures of merit and calculation of the  $E$  map and its interpretation. For difficult structures in polar space groups we have adopted the following enantiomorph-specific *SIMPEL* scheme:

(1) Calculate triplets, quartets and quintets within the group of the strongest unique reflections ( $8n$ , with  $n$  the number of atoms in the asymmetric unit).

(2) Define the origin and give symbols by means of a convergence procedure analogous to that in *MULTAN* (Germain, Main & Woolfson, 1971) using triplets and extreme quartets involving the strongest reflections (e.g. approximately 40).

(3) Extend this small starting set by means of the strongest triplets ( $|\Phi_3| \simeq 0$ ) and, if necessary, by means of positive and negative quartets and quintets in an iterative procedure using very strict acceptance criteria.

(4) Calculate ENQUAC with the group of the most reliable quartets for numerical values of the symbols (e.g. 0,  $\pi/2$ ,  $\pi$ ,  $3\pi/2$ ). The correct symbol combination, in view of (9), is expected to give the lowest ENQUAC value.

(5) Refine the numerical values for the symbols, corresponding to the lowest ENQUAC values, by means of the iterative least-squares method (10).

(6) Calculate ENQUIC with the group of the most reliable quintets only, taking for every symbol numerical values.

(7) Refine the symbol combinations which give the lowest ENQUIC values by means of the least-squares method (21).

(8) Choose the best solutions from steps 5 and 7 on the basis of (10) and (21). Substitute the phases thus obtained for the symbolic phases in the small starting set, which has been generated in step 2. Extend this small starting set and refine the phases with the enantiomorph-specific refinement procedure described in the previous section.

(9) Calculate and interpret  $E$  maps.

### Applications

The enantiomorph-specific *SIMPEL* procedure has been applied to two known structures in space group  $P1$ : (+)-1-menthoxy-methoxy- $\alpha$ -naphthylphenylsilane,  $C_{27}H_{34}O_2Si$  (MENSI) (Kanters & van Veen, 1973) and prostaglandin  $E_2$ ,  $C_{20}H_{32}O_5$  (PGE2) (Edmonds & Duax, 1974a,b). MENSI has been solved with the direct method program *AUDICE* (Spek, 1975) in space group  $P1$ . PGE2 has been solved in space group  $P1$  in an unconventional way, dividing the reflec-

Table 1. Starting set defined by *SIMPEL* for *MENSI*

$h$	$k$	$l$	$E$	(symbolic) phase
-2	1	1	2.81	0
-1	-6	1	2.80	0
-1	-5	1	2.40	0
6	-3	3	2.40	$a$
-6	7	3	2.89	$b$
-1	3	6	2.31	$c$
-2	-6	2	2.34	$d$
-5	11	2	2.57	$e$

tions and the triplets into centrosymmetric and non-centrosymmetric classes (Edmonds & Duax, 1974b).

*MENSI*. 966 triplets with  $E3 > 1.0$ , 1723 quartets (of which all three cross-reflections were measured) with  $E4 > 0.6$  and 377 quintets (of which all ten cross-reflections were measured) with  $E5 > 0.3$  were calculated within the group of the 200 strongest reflections.

*SIMPEL* defined the starting set given in Table 1.

With the strongest triplets and positive quartets the subroutines *STARTZ* and *SYMBAD* could assign 66 reflections a symbolic phase. In addition 31 reflections had more than one indication for their symbolic phase. The figure of merit ENQUAC was calculated for  $4^5 = 1024$  possible solutions on the basis of all quartets with  $E4 > 0.7$ , 47 of which contributed to the summation. In all, 24 small values of ENQUAC were found, one by one identical, because the enantiomorph was not yet defined. In the calculation of ENQUIC, 67 quintets with a lower limit for  $E5$  of 0.35 contributed to the FOM. ENQUIC did not reveal very deep minima, but 40 of them were more pronounced than the others. Only four of the smallest values of both ENQUAC and ENQUIC were found at identical numerical values for the symbols, two of them being unique. These two solutions were refined with the enantiomorph-specific refinement and extension procedure, starting with the set of eight reflections and employing the 690 strongest triplets, the 1126 most reliable quartets and the 212 most reliable quintets. The  $E$  map calculated from one of the two phase sets revealed a large fragment of the structure; the ten highest peaks yielding nine atoms and one spurious peak; the next 25 highest peaks contained 11 atoms, eight spurious peaks and seven atoms of the enantiomorph.

*PGE2*. 877 triplets with  $E3 > 1.0$ , 640 quartets (with three measured cross-reflections) with  $E4 > 1.2$  and 234 quintets (with ten measured cross-reflections) with  $E5 > 0.5$  were calculated within the group of the 200 strongest reflections. The number of quartets and quintets, of which the  $E$  magnitudes of all cross-reflections are known, is small, because many reflections were not measured.

The starting set in Table 2 was found by *SIMPEL*.

After ten cycles of symbolic addition, using the strongest triplets and positive quartets, 60 reflections

Table 2. Starting set defined by SIMPEL for PGE2

<i>h</i>	<i>k</i>	<i>l</i>	<i>E</i>	(symbolic) phase
5	-9	1	3.01	0
5	-10	1	2.82	0
9	0	2	2.40	0
-1	2	1	3.18	<i>a</i>
7	1	2	3.81	<i>b</i>
-1	1	1	2.76	<i>c</i>
7	-4	2	2.81	<i>d</i>
7	4	2	2.75	<i>e</i>

were symbolically phased and for 23 reflections more than one symbolic phase was found. 61 quartets with  $E4 > 1.2$  contributed to ENQUAC. Refinement of the 48 lowest of the 1024 calculated ENQUAC values yielded 18 possible solutions. For the calculation of ENQUIC, 42 quintets with  $E5 > 0.55$  could be used. Refinement of the 48 lowest minima yielded 17 possible solutions; 12 of them were approximately identical to possible solutions of ENQUAC.

These 12 solutions were refined with the enantiomorph-specific refinement and extension procedure. After the refinement the ten strongest peaks in the *E* map of solution number 6 in order of ENQUAC (solution number 3 in order of ENQUIC, solution number 3 in order of ENQUAC + ENQUIC) revealed eight atoms and two atoms of the enantiomorph. The next 20 strongest peaks revealed seven atoms, seven atoms of the enantiomorph and six spurious peaks.

The *E* map of the best solution in order of the combined ENQUAC-ENQUIC figure of merit was not interpretable.

The authors thank Dr C. Stam for a critical reading of the manuscript.

#### References

DETTITA, G. T., EDMONDS, J. W., LANGS, D. A. & HAUPTMAN, H. (1975). *Acta Cryst.* **A31**, 472-479.

*Acta Cryst.* (1979). **A35**, 387-390

- EDMONDS, J. W. & DUAX, W. L. (1974a). *Prostaglandins*, **5**, 275-281.
- EDMONDS, J. W. & DUAX, W. L. (1974b). *Am. Cryst. Assoc. Spring Meeting, Univ. Calif. Abstract A5*.
- FISCHER, J., HANCOCK, J. & HAUPTMAN, H. (1970). Report No. 7132. Naval Research Laboratory, Washington, D.C.
- GERMAIN, G., MAIN, P. & WOOLFSON, M. M. (1971). *Acta Cryst.* **A27**, 368-376.
- GIACOVAZZO, C. (1975). *Acta Cryst.* **A31**, 252-259.
- GIACOVAZZO, C. (1977). *Acta Cryst.* **A33**, 527-531.
- GILMORE, C. (1977). *Acta Cryst.* **A33**, 712-716.
- GILMORE, C. (1978). Private communication.
- HAUPTMAN, H. (1974). *Acta Cryst.* **A30**, 472-476.
- HAUPTMAN, H. (1975). *Acta Cryst.* **A31**, 680-687.
- HAUPTMAN, H., FISCHER, J., HANCOCK, H. & NORTON, D. A. (1969). *Acta Cryst.* **B25**, 811-814.
- HAUPTMAN, H. (1978). Private communication.
- HAUPTMAN, H. & FORTIER, S. (1977). *Acta Cryst.* **A33**, 575-580.
- KANTERS, J. A. & VAN VEEN, A. M. (1973). *Cryst. Struct. Commun.* **2**, 261-265.
- KARLE J. & HAUPTMAN, H. (1958). *Acta Cryst.* **11**, 264-269.
- LESSINGER, L. (1976). *Acta Cryst.* **A32**, 538-550.
- MAIRESSE, G. & DRACKE, M. (1978). *Acta Cryst.* **B34**, 1771-1776.
- OVERBEEK, A. R., VAN DER PUTTEN, N., OLTHOF, G. J. & SCHENK, H. (1977). *SIMPEL* manual, Amsterdam.
- PUTTEN, N. VAN DER (1978). In preparation.
- PUTTEN, N. VAN DER & SCHENK, H. (1976). *K. Ned. Akad. Wet. Proc.* **B79**, 344-352.
- PUTTEN, N. VAN DER & SCHENK, H. (1977). *Acta Cryst.* **A33**, 856-858.
- SCHENK, H. (1972). *Acta Cryst.* **A28**, 412-422.
- SCHENK, H. (1974). *Acta Cryst.* **A30**, 477-481.
- SCHENK, H. (1975). *Acta Cryst.* **A31**, 259-263.
- SCHENK, H. (1976). Proc. Buffalo Direct Methods Symp., 1976.
- SCHENK, H. & PUTTEN, N. VAN DER (1977). *Acta Cryst.* **A33**, 368-372.
- SINT, L. & SCHENK, H. (1975). *Acta Cryst.* **A31**, S22.
- SPEK, A. L. (1975). Thesis, Utrecht.

## Electron Microscopy Studies of Mo<sub>3</sub>CoSi

BY LARS STENBERG

*Inorganic Chemistry 2, Chemical Center, PO Box 740, S-220 07 Lund, Sweden*

(Received 17 June 1978; accepted 27 November 1978)

#### Abstract

Crystals of Mo<sub>3</sub>CoSi were studied with a high-resolution electron microscope. Planar defects of different kinds were easily and frequently observed. The structure of one kind of defect can be derived with crystallographic shear operations.

0567-7394/79/030387-04\$01.00

#### Introduction

The principles of formation of the so-called tetrahedrally close-packed alloy structures, involving different crystallographic operations and intergrowth, have recently been described by Andersson (1978). In this way it is possible to describe and predict defects and a

© 1979 International Union of Crystallography